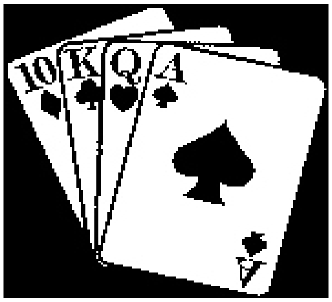


Probability Theory
Lesson 10
Probability Sample space

10.1 - What is a Probability Sample Space?

A probability sample space is a sample space where each event **E** in the sample space has a number P(**E**) associated with it. The study of probability sample spaces allows for a systematic approach for solving probability problems. For the value P(**E**) we have the following laws:

1. $0 \leq P(\mathbf{E}) \leq 1$
2. $P(\mathbf{S}) = 1$
3. $P(\mathbf{E}') = 1 - P(\mathbf{E})$
4. $P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B})$

From these laws, one can show the following laws hold truth:

5. $P(\phi) = 0$
6. $P(\mathbf{E}) = 1 - P(\mathbf{E}')$
7. If $\mathbf{A} \cap \mathbf{B} = \phi$ then $P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$
8. $P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cup \mathbf{B})$

To find the probability of an event P(**E**), the following systematic approach should be followed:

I. Express the event **E** as a Boolean expression of the events given.

II. Apply the above laws, to the Boolean expression.

The following examples and problems show how these laws are used to solve probability problems.

10.1 - Example 1: Ms. Jones receives at most 100 phone calls a day. Assume that the probability that she will receive at least 10 phone calls a day is 1/4.

(a). Find the probability that she will receive at most 9 calls a day.

(b). Interpret the meaning of the value P(**E**).

Solutions:

➤ **(a)**.
I. First write out the sample space: **S** = {0, 1, 2, 3,..., 100}.

The event that she will receive at least 10 phone calls is **M** = {10, 11, 12, 13,..., 100}.

The event that she will receive at most 9 calls a day is **E** = {0, 1, 2,..., 9}.

We can write **E** in terms of **M** by seeing that **E** = **M**′ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

II. To find the probability of P(**E**), we use the formula

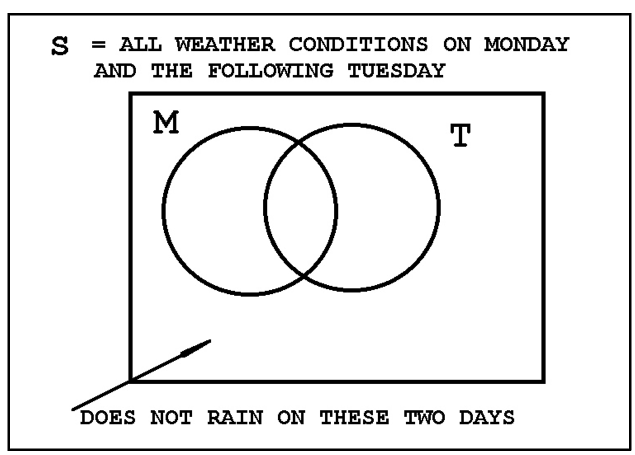
$$P(E) = P(M') = 1 - P(M) = 1 - \frac{1}{4} = \frac{3}{4}.$$

➤ **(b)**.
On any day, there is a 75% chance that Ms. Jones will receive less than 10 calls a day.

10.1 - Example 2: Records of rain fall in Southern Iowa shows that the probability of rain fall on Monday is 0.65, on the following Tuesday is 0.70 and on both Monday and Tuesday, the chance of rain fall is 0.45.

(a). Find the probability that it will rain on Monday or the following Tuesday.

(b). Interpret the meaning of the value P(**E**).



Solutions:

➤ **(a)**.
I. The sample space **S** consists of all possible weather conditions for Monday and the following Tuesday.

Let **M** represent the event that it will rain on Monday; Let **T** represent the event that it will rain on the following Tuesday; let **E** be the event that it will rain on Monday or the following Tuesday.

Therefore, **E** = **M**∪**T**.

II. From the example, we have P(**M**) = 0.65, P(**T**) = 0.70 and P(**M**∩**T**) = 0.45.

From Step 2 and law 4,

P(**E**) = P(**M**∪**T**) = P(**M**) + P(**T**) - P(**M**∩**T**) = 0.65 + 0.70 - 0.45 = 0.90.

➤ **(b)**.
For any week, there is a 90% chance that it will rain on Monday or Tuesday.

10.1 - Example 3: An urn contains red, white and blue marbles. A marble is selected at random. The chance of selecting a red marble is 1/3 and a white marble is 1/4. Find the probability that

(a). a red or white marble is selected.

(b). a blue marble is selected.

Solutions:

I. S = {red marble,white marble,blue marble}

➤ **(a)**.
Let **R** stand for the event a red marble is selected; Let **W** stand for the event a white marble is selected. The event **E** that a red or white marble is selected can be written as **E** = **R**∪**W**.

Since both a red and white marble cannot be both selected, **R**∩**W** = ϕ

II. Using law 7, we have P(**E**) = P(**R**∪**W**) = P(**R**) + P(**W**) = $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.

➤ **(b)**.
I. The event **E,** a blue marble is selected can be written

E = **W**′∩**R**′ = (**W**∪**R**)′, neither a white marble nor red marble was selected.

II. From laws 3 and 7, P(**E**) = P((**W**∪**R**)′) = 1 - P(**W**∪**R**) = $1 - \frac{7}{12} = \frac{5}{12}$.

10.1 - Example 4: A computer generates random whole numbers from 1 to 10. A study showed that the chance it will generate a random number from 1 to 5 is 0.60 and from 3 to 10 is 0.70. Find the probability that it will generate random numbers from 3 to 5 inclusive.

Solution:

I. The sample space is **S** = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Let **A** = {1, 2, 3, 4, 5}; **B** = {3, 4, 5, 6, 7, 8, 9, 10}.

The event **E =** {3, 4, 5} = **A**∩**B**.

II. Using law 8, P(**E**) = P(**A**∩**B**) = P(**A**) + P(**B**) - P(**A**∪**B**).

Since **A**∪**B** = {1, 2, 3, 4, 5}∪{3, 4, 5, 6, 7, 8, 9, 10} = **S**, we have P(**A**∪**B**) = P(**S**) = 1.

From the example, P(**A**) = 0.60, P(**B**) = 0.70 .

Therefore, P(**E**)= P(**A**∩**B**) = P(**A**) + P(**B**) - P(**A**∪**B**) = 0.60 + 0.70 - 1 = 0.30 .

10.1 - Example 5: Ms. Jones receives at least 10 calls a day.

Let **A** be the event she receives at most 15 calls a day;
Let **B** be the event she receives at least 12 calls a day;
Let **C** be the event she receives between 15 and 20 calls a day.

Assume: P(**A**) = 0.60, P(**B**) = 0.85, P(**C**) = 0.25, P(**A**∪**C**) = 0.40 .

Find the probability of the event **E** that

(a). she receives at least 16 calls a day.

(b). she receives 10 or 11 calls a day (inclusive).

(c). she receives between 12 and 15 calls a day (inclusive).

(d). she receives exactly 15 calls a day.

(e). she receives at least 16 calls a day or less than 12 calls a day.

Solutions:

➤ **(a)**.
I. The sample space **S** = {10, 11, 12,...}.

A = {10, 11, 12, 13, 14, 15} and **A**′ = { 16, 17, 18,...}

E = {16, 17,...}

Therefore, **E** = **A**′.

II. P(**E**) = P(**A**′) = 1 - P(**A**) = 1 - 0.60 = 0.40 .

➤ **(b)**.
I. B = {12, 13, 14,...}, **B**′ = {10, 11}

Therefore, **E** = **B**′.

II. P(**E**) = P(**B**′) = 1 - P(**B**) = 1 - 0.85 = 0.15 .

➤ **(c)**.
I. A = {10, 11, 12, 13, 14, 15}, **B** = {12, 13, 14, 15, 16,...},

E = {12, 13, 14, 15}

A∩**B** = {12, 13, 14, 15}

Therefore, **E** = **A**∩**B**.

II. Using law 8, P(**E**) = P(**A**∩**B**) = P(**A**) + P(**B**) - P(**A**∪**B**) = 0.60 + 0.85 - P(**A**∪**B**).

Since **A**∪**B** = **S** , P(**E**) = P(**A**∩**B**) = P(**A**) + P(**B**) - P(**A**∪**B**) = 0.60 + 0.85 - P(**S**) = 0.60 + 0.85 - 1 = 0.45 .

➤ **(d)**.
I. A = {10, 11, 12, 13, 14, 15}, **C** = {15, 16, 17, 18, 19, 20}

E = A∩**C** = {15}

II. Using law 8,

P(**E**) = P(**A**∩**C**) = P(**A**) + P(**C**) - P(**A**∪**C**) = 0.60 + 0.25 - 0.40 = 0.45 .

➤ **(e)**.
I. A′ = {16, 17, ...}, **B**′ = {10,11}

E = **A**′∪**B**′

II. Since **A**′∩**B**′ **=** ϕ , we use law 7:

P(**E**) = P(**A**′∪**B**′) = P(**A**′) + P(**B**′)

By law 3,

P(**A**′) = 1 - P(**A**) = 1 - 0.60 = 0.4

P(**B**′) = 1 - P(**B**) = 1 - 0.85 = 0.15

Therefore, P(**E**) = P(**A**′) + P(**B**′) = 0.4 + 0.15 = 0.55 .

Solved Problems

10.1 - Solved Problem 1: Ms. Jones receives at most 100 phone calls a day. Assume that the probability that she will receive at least 1 phone call is 9/10.

(a). Find the probability that she will receive no calls.

(b). Interpret the meaning of the value P(**E**).

Solutions:

➤ **(a)**.
I.: **S** = {0, 1, 2, 3,..., 100}

E = {0}

Let **M** = {1, 2, 3,..., 100}, **M′** = {0}.

Therefore, **E** = **M′**.

II. P(**E**) = P(**M′**) = 1 - P(**M**) = 1 - $\frac{9}{10}$ = $\frac{1}{10}$

➤ **(b)**.
On any day, there is a 10% chance that Ms. Jones will receive no call a day.

10.1 - Solved Problem 2: A recent survey showed that 25% of all families in New York own dogs, 40% own cats and 10% own both dogs and cats.

(a). If a family from New York is selected at random, find the probability that the family owns dogs or cats.

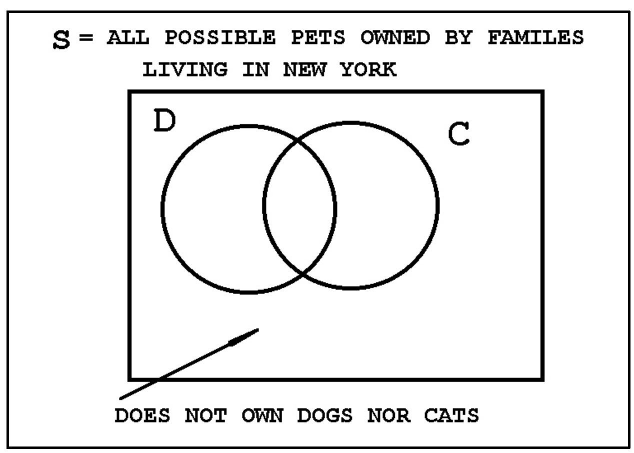
(b). Assume a family from New York is randomly selected. Interpret the meaning of the value P(**E**).

Solutions:

➤ **(a)**.
I. The sample space consists of all possible pets families in New York own.

Let **D** represent the event that the family owns dogs; Let **C** represent the event that the family owns cats ; let **E** be the event that the family owns dogs or cats.

Therefore, **E** = **D**∪**C**.



II. From the example, we have P(**D**) = 0.25, P(**C**) = 0.40 and P(**D**∩**C**) = 0.10.

From Step 2 and law 4,

P(**E**) = P(**D**∪**C**) = P(**D**) + P(**C**) - P(**D**∩**C**) =

0.25 + 0.40 - 0.10 = 0.55.

➤ **(b)**. There is a 55% chance that the family will own a dog or a cat.

10.1 -Solved Problem 3: A statistics class has freshmen, junior, and senior students. 70% of the students are freshmen and 20% are seniors. A student is selected at random. Find the probability that (a). a freshman or senior is selected.

(b). a junior is selected.

Solutions:

I. S = {freshmen, junior, senior}

➤ **(a)**.

Let **F** stand for the event a freshman is selected; Let **S** stand for the event a senior is selected. The event E, a freshman or senior student is selected can be written as **E** = **F**$\cup$**S**.

Since both a freshman and senior cannot be both selected, **F**$\cap$**S** = ϕ

II. Using law 7, we have P(**E**) = P(**F**$\cup$**S**) = P(**F**) + P(**S**) = 0.70 + 0.20 = 0.90 .

➤ **(b)**.

I. The event **J** a junior can be written **J** = **F**$'\cap$**S**$'$= (**F**$\cup$**S**)$'$, neither a freshman nor senior was selected.

II. From law 3 and law 7, P(**J**) = P[(**F**$\cup$**S**)$'$] = 1 - P(**F**$\cup$**S**) = 1 - 0.90 = 0.10.

10.1 - Solved Problem 4: A machine produces an important part to an automobile transmission. Each hour, it produces 10 such parts. A study showed that each hour, the chance it will produce at most 5 defective parts is 0.35 and more than 3 defective parts is 0.65. Find the probability that it will produce 4 or 5 defective parts.

Solution:

I. The sample space is **S** = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. This sample space represents the possible number of defective parts produced per hour.

Let **A** = {0, 1, 2, 3, 4, 5}, **B** = {4, 5, 6, 7, 8, 9, 10}.

The event **E** = {4, 5} = **A**$\cap$**B**.

II. Using law 8, P(**E**)= P(**A**$\cap$**B**) = P(**A**) + P(**B**) - P(**A**$\cup$**B**).

Since **A**$\cup$**B** = {0, 1, 2, 3, 4, 5}$\cup${4, 5, 6, 7, 8, 9, 10} = **S**, we have P(**A**$\cup$**B**) = P(**S**) = 1 .

From the example, P(**A**) = 0.35, P(**B**) = 0.65 .

Therefore, P(**E**)= P(**A**$\cap$**B**) = P(**A**) + P(**B**) - P(**A**$\cup$**B**) = 0.35 + 0.65 - 1 = 0 .

10.1 - Solved Problem 5: The California Highway Patrol estimates that on a certain section of the San Diego freeway, highway patrolmen issue at least 100 speeding tickets a day.

let **A** be the event that at most 150 tickets are issued each day;
Let **B** be the event that at least 120 tickets are issued each day;

Let **C** be the event that between 150 and 200 tickets are issued each day.

Assume: P(**A**) = 0.75, P(**B**) = 0.90, P(**C**) = 0.12, P(**A**∪**C**) = 0.55 .

Find the probability of the event **E** that

(a). at least 151 tickets are issued a day.

(b). less than 120 tickets are issued a day.

(c). between 120 and 150 tickets are issued a day (inclusive).

(d). exactly 150 tickets are issued.

(e). more than 150 tickets are issued each day or less than 120 tickets are issued each day.

Solutions:

➤ **(a)**.
I. The sample space **S** = {100, 101, 102,...}.

A = {100, 101,..., 150}, **A**′ = {151, 152, 153,...}

E = {151, 152, 153,...}

Therefore, **E** = **A**′

II. P(**E**) = P(**A**′) = 1 - P(**A**) = 1 - 0.75 = 0.25

➤ **(b)**.
I. B = {120, 121, 122,...}, **B**′ = {100, 101,..., 119}

E = {100, 101,..., 119}

Therefore, **E** = **B**′.

II. P(**E**) = P(**B**′) = 1 - P(**B**) = 1 - 0.90 = 0.10

➤ **(c)**.
I. B = {120, 121, 122,...}, **A** = {100, 101,..., 150}

E = {120, 121,..., 150}

A∩**B** = {120, 121,..., 150}

Therefore, **E = A**∩**B**.

II. Using law 8, P(**E**) = P(**A**∩**B**) = P(**A**) + P(**B**) - P(**A**∪**B**) = 0.75 + 0.90 - P(**A**∪**B**).

Since $\mathbf{A} \cup \mathbf{B} = \mathbf{S}$, $P(\mathbf{E}) = P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cup \mathbf{B}) = 0.75 + 0.90 - P(\mathbf{S}) = 0.75 + 0.90 - 1 = 0.65$.

➤ **(d)**.
I. $\mathbf{A} = \{100, 101, 102, ..., 150\}$, $\mathbf{C} = \{150, 151, ..., 200\}$

$\mathbf{E} = \mathbf{A} \cap \mathbf{C} = \{150\}$

II. Using law 8,

$P(\mathbf{E}) = P(\mathbf{A} \cap \mathbf{C}) = P(\mathbf{A}) + P(\mathbf{C}) - P(\mathbf{A} \cup \mathbf{C}) = 0.75 + 0.12 - 0.55 = 0.32$.

➤ **(e)**.
I. More than 150 tickets are issued each day: $\mathbf{A}' = \{151, 152, ...\}$.

Less than 120 tickets are issued each day: $\mathbf{B}' = \{100, 101, ..., 119\}$.

$\mathbf{E} = \mathbf{A}' \cup \mathbf{B}'$

II. Since $\mathbf{A}' \cap \mathbf{B}' = \phi$, we use law 7:

$P(\mathbf{E}) = P(\mathbf{A}' \cup \mathbf{B}') = P(\mathbf{A}') + P(\mathbf{B}')$.

By law 3,

$P(\mathbf{A}') = 1 - P(\mathbf{A}) = 1 - 0.75 = 0.25$

$P(\mathbf{B}') = 1 - P(\mathbf{B}) = 1 - 0.90 = 0.10$

Therefore, $P(\mathbf{E}) = P(\mathbf{A}' \cup \mathbf{B}') = P(\mathbf{A}') + P(\mathbf{B}') = 0.25 + 0.10 = 0.35$.

Unsolved Problems with Answers

10.1 - Problem 1: Ms. Romano's Latin class has 50 students. Assume that the probability that she will have at most 48 students at the end of the semester is 0.72.

(a). Find the probability that at the end of the semester, at least 49 students will finish the class.

(b). Interpret the meaning of the value $P(\mathbf{E})$.

Answers:

➤ **(a)**. 0.28

➤ **(b)**. There is a 28% chance that at the end of the semester Ms. Romano's Latin class will have at least 49 students remaining.

⇑ *Refer back to* **10.1 - Example 1 & 10.1 - Solved Problem 1**.

10.1 - Problem 2: A computer manufacturer recently received a large shipment of computer chips. The chance that at least 10 chips are defective is 0.15; that between 5 and 20 chips are defective is 0.30 and between 10 and 20 chips are defective is 0.05.

(a). Find the probability that the box contains more than 4 defective chips.

(b). Interpret the value P(**E**).

Answers:

➤ **(a)**. 0.40

➤ **(b)**. There is a 40% chance that the box contains more than 4 defective chips.

⇑ *Refer back to* **10.1 - Example 2 & 10.1 - Solved Problem 2**.

10.1 - Problem 3: Jill is attending a local community college. To finish her degree she needs to take only one class from the following classes: American history, British history or Ancient Greek history. The chance that she will take American history is 0.53 and Ancient Greek history is 0.25. Find the probability that she will take

(a). British History.

(b). British History or Ancient Greek History.

Answers:

➤ **(a)**. 0.22

➤ **(b)**. 0.47

⇑ *Refer back to* **10.1 - Example 3 & 10.1 - Solved Problem 3**.

10.1 - Problem 4: A study of the World Historical Club estimates that a typical American history text book has many factual errors. The study further showed that the chance of at least 50 errors is 0.35 and the chance of less than 60 errors is 0.70. Find the probability that an American history book contains between 50 and 59 historical errors. (inclusive).

Answer: 0.05

⇑ *Refer back to* **10.1 - Example 4 & 10.1 - Solved Problem 4**.

10.1 - Problem 5: Johnny Roadster finished third in a local stock car race. The following events relate to his speed at the finish line where **S** = {100, 101, 102,..., 170}.

Let **A** be the event his speed was at most 160 mph.
Let **B** be the event his speed was at least 150 mph.
Let **C** be the event his speed was between 160 and 170 mph.

Assume: P(**A**) = 0.90, P(**B**) = 0.85, P(**C**) = 0.25.

Find the probability of the event **E** that his speed was

(a). more than 160 mph.

(b). less than 150 mph.

(c). between 150 and 159 mph (inclusive).

(d). exactly 160 mph.

(e). less than 150 mph or between 160 and 170 mph.

Answers:

➤ **(a)**. 0.10

➤ **(b)**. 0.15

➤ **(c)**. 0.60

➤ **(d)**. 0.15

➤ **(e)**. 0.40

⇑ *Refer back to* **10.1 - Example 5 & 10.1 - Solved Problem 5.**

10.2 - Defining the Probability of Events when Elements of the Sample Space are equally likely to Occur.

Assume an experiment is to be defined where the sample space is finite and each element of the sample space has an equal chance to occur. The probability of an event **E** is defined as

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}}.$$

Since each element of the event E has equal chance of being selected, this ratio gives the best numerical value to represent the probability that the event **E** will occur.

10.2 - Example 1: A fair coin is tossed once. Find the probability that a head occurs.

Solution:

I. The sample space is **S** = {h, t} where h stands for heads occurs and t for tails occurs.

Let **E** = {t}.

#**E** = 1 and #**S** = 2.

II. Since both elements t and h are equally likely to occur, P(**E**) = $\frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{1}{2}$.

10.2 - Example 2: A fair coin is tossed twice. Find the following probability that:

(a). a head and tail occurs.

(b). at least one head occurs.

(c). two sides are the same.

Solutions:

I. S = {(h,h), (h,t), (t,h), (t,t)}

➤ **(a)**.
E = {(h,t), (t,h)}

#**E** = 2 and #**S** = 4.

II. Since each element of the sample space has equal chance to occur,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{2}{4} = \frac{1}{2}.$$

➤ **(b)**.
I. The event "at least one heads occurs" can be written **E** = {(h,t), (t,h), (h,h)}

II. Since each element of the sample space has equal chance to occur,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{3}{4}.$$

➤ **(c)**.
I. The event "two sides are the same" can be written

E = {(t,t), (h,h)}

II. $P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{2}{4} = \frac{1}{2}.$

10.2 - Example 3: A local reading club has 25 members. A survey was taken as to their reading preferences. The following is the result of this survey:

Eleven members like historical novels.

Ten members like mysteries.

Ten members like romance novels.

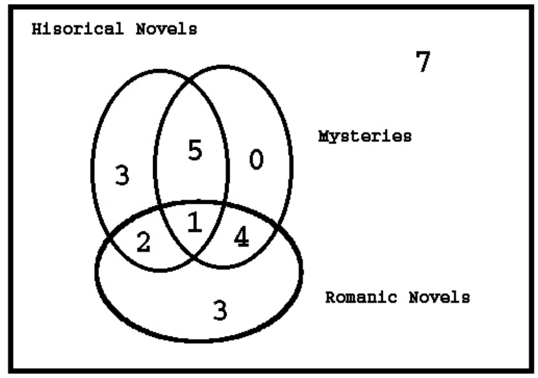
Three members like history and romance novels.

Five members like mysteries and romance novels.

Six members like history novels and mysteries.

One member like all three.

Assume a member is selected at random. Find the probability that the member:



(a). only likes history novels.

(b). likes mysteries and romance but does not like history novels.

(c). only likes one of these types of novels.

(d). likes history or romance novels but does not like mysteries.

(e). does not like any of these types of books.

Solutions:

We use the above Venn Diagram.

➤ **(a)**.
Here we know that #**S** = 25. The event **E** "that the member only likes historical novels" has cardinality

#**E** = 3. Since each member of the club has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{3}{25}.$$

➤ **(b)**.
The event **E** "likes mysteries and romance but does not like history novels" has cardinality #**E** = 4 . Since each member of the club has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#E}{\#S} = \frac{4}{25}.$$

➤ **(c)**.
The event **E** "only likes one of these types of books" has cardinality #**E** = 3 + 3 + 0 = 6. Since each member of the club has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#E}{\#S} = \frac{6}{25}.$$

(d).
The event **E**: "likes history or romance novels but does not like mysteries" has cardinality #**E** = 3 + 2 + 3 = 8

Since each member of the club has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{8}{25}.$$

(e).
The event **E**: "does not like any of these types of books" has cardinality #**E** = 7. Since each member of the club has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{7}{25}.$$

10.2 - Example 4: One card is drawn from any ordinary deck of cards. Find the probability that it is a king or queen.

Solution:

I. K: The event a king is drawn.
Q: The event a queen is drawn.
E: The event a king or queen is drawn.

$\mathbf{E} = \mathbf{K} \cup \mathbf{Q}$

II. P(**K**) = 4/52

P(**Q**) = 4/52

Since $\mathbf{K} \cap \mathbf{Q} = \phi$ apply law 7: $P(\mathbf{E}) = P(\mathbf{K} \cup \mathbf{Q}) = P(\mathbf{K}) + P(\mathbf{Q}) = 4/52 + 4/52 = 8/52$.

Solved Problems

10.2 - Solved Problem 1: An urn contains three marbles colored red, white and blue. A marble is selected at random. Find the probability that a red or white marble is selected.

Solution:

I. The sample space is **S** = {r, w, b} where r stands for a red marble is being selected, w stands for a white marble being selected and b stands for a blue marble being selected.

Let **E** = {w, r}.Then **#E =** 2 and #**S** = 3.

II. Since each marble in the urn has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{2}{3}.$$

10.2 - Solved Problem 2: A die is tossed twice. Find the following probability that:

(a). a one and six occurs.

(b). at least one three occurs.

(c). both numbers are the same.

Solutions:

I. **S** = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5) ,(2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

➤ **(a)**.
The number 1 can occur on the first toss and 6 on the second toss **or** the number 6 can occur on the first toss and 1 on the second toss. Therefore, **E** = {(1,6), (6,1)}, #**E** = 2 and #**S** = 36.

II. Since each pair of numbers has equal chance of occurring,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{2}{36}.$$

➤ **(b)**.
II. The event "at least one three occurs" can be written

E = {(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (1,3), (2,3), (4,3), (5,3), (6,3)}.

#**E** = 11.

II. Since each pair of numbers has equal chance of occurring,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{11}{36}.$$

➤ **(c)**.
I. The event "both numbers are the same" can be written

E = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}.

#**E** = 6

II. Since each pair of numbers has equal chance of occurring,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{6}{36}.$$

10.2 - Solved Problem 3: A recent survey of 150 men was taken to find out their participation in the following sports: football, baseball, and ice hockey. From the survey, the following information was given:

13 men participated in all three sports.

20 men participated in football and baseball.

30 men participated in football and ice hockey.

22 men participated in ice hockey and baseball.

45 men participated in football.

48 men participated in ice hockey.

30 men participated in baseball.

Assume a man is selected at random. Find the probability that he:

(a). only likes football.

(b). likes ice hockey and baseball but does not like football.

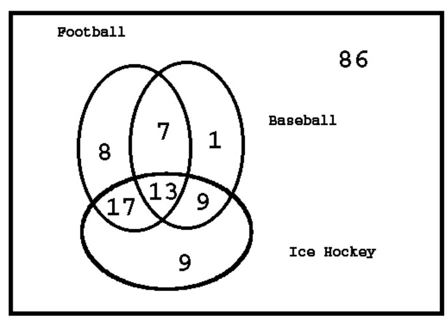
(c). only likes one of these sports.

(d). likes football or baseball but does not like ice hockey.

(e). does not like any of these sports.

Solutions:

We use the Venn Diagram



➤ **(a)**.
From the statement of the problem #**S** = 150. The event **E** "that he only like football" has cardinality #**E** = 8. Since each man has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{8}{150}.$$

➤ **(b)**.
The event **E** "he likes ice hockey and baseball but does not like football" has cardinality #**E** = 9. Since each man has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{9}{150}.$$

➤ **(c)**.
The event **E** "he likes only one of these sports" has cardinality #**E** = 8 + 1 + 9 = 18. Since each man has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{18}{150}.$$

➤ **(d)**.
The event **E** "likes football or baseball but does not like ice hockey has cardinality " #**E** = 8 +7 + 1 = 16 . Since each man has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{16}{150}.$$

➤ **(e)**.
The event **E** "does not like any of these sports" has cardinality #**E** = 86. Since each man has equal chance of being selected,

$$P(\mathbf{E}) = \frac{\#\mathbf{E}}{\#\mathbf{S}} = \frac{86}{150}.$$

10.2 - Solved Problem 4: One card is drawn from an ordinary deck of cards. Find the probability that it is a king or diamond.

Solution:

I. K: The event that a king is drawn.
D: The event that a queen is drawn.
E: The event that a king or diamond is drawn.

$\mathbf{E} = \mathbf{K} \cup \mathbf{D}$

II. P(**K**) = 4/52

P(**D**) = 13/52

Since $\mathbf{K} \cap \mathbf{D} \neq \phi$, apply law 4: $P(\mathbf{E}) = P(\mathbf{K} \cup \mathbf{D}) = P(\mathbf{K}) + P(\mathbf{Q}) = P(\mathbf{K} \cap \mathbf{D}) = 4/52 + 13/52 - 1/52 = 16/52$.

Unsolved Problems with Answers

10.2 - Problem 1: A card is randomly selected from an ordinary deck of cards. Find the probability that a diamond is selected.

Answer:

$$\frac{13}{52}$$

⇑ *Refer back to* **10.2 - Example 1 & 10.2 - Solved Problem 1.**

10.2 - Problem 2: A coin is tossed three times. Find the following probability that:

(a). two heads and a tail occurs.

(b). at least one tail occurs.

(c). all sides are the same.

Answers:

➤ **(a)**. $\frac{3}{8}$

➤ **(b)**. $\frac{7}{8}$

➤ **(c)**. $\frac{2}{8}$

⇑ *Refer back to* **10.2 - Example 2 & 10.2 - Solved Problem 2.**

10.2 - Problem 3: A survey of 200 farmers produced the following results:

120 raised chickens.
104 raised hogs.
94 raised sheep.
66 raised both chickens and hogs.
57 raised both chickens and sheep.
47 raised both hogs and sheep.
10 raised all three.

A farmer is selected at random. Find the probability that the farmer

(a). only raises sheep.

(b). raises sheep and hogs but not chickens.

(c). only raises one of these animals.

(d). raises sheep or hogs but not chickens.

(e). does not raise any of these animals.

Answers:

➤ **(a)**. 0

➤ **(b)**. $\frac{37}{200}$

➤ **(c)**. $\frac{8}{200}$

➤ **(d)**. $\frac{38}{200}$

➤ **(e)**. $\frac{42}{200}$

⇑ *Refer back to* **10.2 - Example 3 & 10.2 - Solved Problem 3.**

10.2 - Problem 4: One card is drawn from an ordinary deck of cards. Find the probability that it is a diamond or face card.

Answer:

22/52

⇑ *Refer back to* **10.2 - Example 4 & 10.2 - Solved Problem 4**.

Supplementary Problems

A roulette wheel has numbers 0,1,2,.., 36 and 00. Each time the wheel is spun, a ball falls on one of these numbers. Assume we spin the wheel three times. Find the following probabilities:

1. 00 appears all three times.

2. all the numbers are the same.

3. only two fives appear.

A survey of local teenagers produced the following results:

54 like Burger King.
28 like McDonald's.
36 like Wendy's.
13 like McDonald's and Wendy's.
18 like Wendy's and Burger King.
12 like McDonald's and Burger King.
5 like all three.
20 do not like any of the three.

Assume a teenager is selected at random. Find the probability

4. that the teenager likes McDonald's but does not like Burger King.

5. that the teenager likes McDonald's or does not like Burger King.

6. Given that the teenager likes Burger King and Wendy's, the teenager also likes McDonald.

7. Given that the teenager likes Burger King or Wendy's, the teenager does not like McDonald's

A die is tossed twice. Find the probability

8. that the sum of the numbers is either greater than 10 or equal to four.

9. the sum of the numbers is between 2 and 4 inclusive.

In September, 1988, the House of Representatives voted on an amendment requiring life imprisonment for drug-related murders. Results of the vote were reported as shown below:

	YEA	NAY	DID NOT VOTE	Total
Democrat	153	83	19	255
Republican	169	0	8	177
Total	322	83	27	432

10. What is the probability that a randomly selected representative voted for the amendment?

11. What is the probability that a randomly selected representative is both a Democrat and voted in favor of the amendment?

12. Given that a representative voted for the amendment, what is the probability that he/she is a Democrat?

13. Show the formula

a. $P(\mathbf{A}\cup\mathbf{B}\cup\mathbf{C}) = P(\mathbf{A}) + P(\mathbf{B}) + P(\mathbf{C}) - P(\mathbf{B}\cap\mathbf{C}) - P(\mathbf{A}\cap\mathbf{B}) - P(\mathbf{A}\cap\mathbf{C}) + P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{A}\cap\mathbf{C})\}$.

b. A game of pool is being played. There are 10 balls on the table, each numbered 1 through 10 respectively. Assume a ball is randomly shot into a pocket. Use the above formula to find the probability that the ball is an odd number or a number between 5 to 9 (inclusive) or a number between 3 to 7 (inclusive).

14. Show that $P(\mathbf{A}_1\cup\mathbf{A}_2\cup\mathbf{A}_3\cup...\cup\mathbf{A}_n) \leq P(\mathbf{A}_1) + P(\mathbf{A}_2) + P(\mathbf{A}_3) + ... + P(\mathbf{A}_n)$.

15. Assume an experiment creates a sample space **S** and $P(\mathbf{A}) = 0.60$, $P(\mathbf{B}) = 0.5$ and $P(\mathbf{A}\cup\mathbf{B}) = 0.65$. Using combinations of **A**, **B**,$\cap$,$\cup$,$'$ find the probabilities of all possible non-empty events.(See Supplementary problem 1 in Lesson 3, Sets).

16. If $\mathbf{A} \subseteq \mathbf{B}$ show $P(\mathbf{A}) \leq P(\mathbf{B})$.

17. If $P(\mathbf{A}) \leq P(\mathbf{B})$ show $P(\mathbf{B}') \leq P(\mathbf{A}')$.

18. For events **A**,**B**,**C**, show $P(\mathbf{A}\cap\mathbf{C}') \leq P(\mathbf{A}\cap\mathbf{B}') + P(\mathbf{B}\cap\mathbf{C}')$.

19. For events **A**, **B**, **C**, show $P(\mathbf{A}\cap\mathbf{B}) \leq P(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})$.

20. For the two events **A** and **B**, show that $P(\mathbf{A}) + P(\mathbf{B}) - 2P(\mathbf{A}\cap\mathbf{B})$ is the probability that only 1 of these events

occurs.

21. Assume P(**A**) = 1/3 and P(**B**) = 3/4. Show $1/12 \le$ P(**A**∩**B**) $\le 1/3$.

22. Explain why the following statements are true or false.

a. For all **A**,**B**, if P(**A**) = 0, then **A** = ϕ.

b. For a sample space if P(**A**) = 0.76 and P(**B**) = 0.92 can P(**A**∪**B**) $\le$ 1?

c. For a sample space, if P(**A**∩**B**) > 0, can P(**A**) = 0.76 and P(**B**) = 0.92?

d. For all **A**,**B** , if **A** ⊆**B** and P(**A**) = P(**B**) then **A** = **B**

e. If **A**∩**B** = ϕ, can P(**A**) = 0.76 and P(**B**) = 0.92?

f. Can P(**A**) = 0.26 and P(**B**) = 0.52 and P(**A**∩**B**) = 0.80?

23. Assume P(**A**) = P(**B**) = P(**C**) = 0.50, P(**A**∩**B**) = P(**A**∩**C**) = P(**B**∩**C**) = 0.25, P(**A**∩**B** ∩**C**) = 0.125.

a. Show **A**∩**B** = (**A**∩**B** ∩**C**′)∪(**A**∩**B** ∩**C**) (Hint: use Anti-symmetric law: if **M** ⊆ **N** and **N** ⊆ **M** then **M = N**).

b. Show **A** = (**A**∩**B**′∩**C**′)∪(**A**∩(**B** ∪ **C**).

c. Find P(**A**∩**B**∩**C**′), P(**A**∩**B**′∩**C**), P(**A**′∩**B**∩**C**), P(**A**∩**B**′∩**C**′), P(**A**′∩**B**∩**C**′), P**A**′∩**B**′∩**C**), P(**A**′∩**B**′∩**C**′).

24. Show $P(\mathbf{A}_1 \cup \mathbf{A}_2 \cup ... \cup \mathbf{A}_n) = 1 - P(\mathbf{A}_1' \cap \mathbf{A}_2' \cap ... \cap \mathbf{A}_n')$.

For the following problems assume a sample space **S** and various events **A, B, C** etc.

25. Assume **A**∩**B** = **A**∩**C** = **B**∩**C** = ϕ. Show P(**A**∪**B**∪**C**) = P(**A**) + P(**B**) + P(**C**).

26. Assume **A**∩(**B** ∪**C**) = ϕ. Show P(**A**∪**B**∪**C**) = P(**A**) + P(**B**) + P(**C**) - P(**B**∩**C**).

27. Show P(**B**∩**C**) + P(**B**′∩**C**) + P(**B**∩**C**′) + P(**B**′∩**C**′) = 1.

28. Show P(**A** ∩**B**∩**C**) + P(**A** ∩**B**′∩**C**) + P(**A** ∩**B**∩**C**′) + P(**A**∩**B**′∩**C**′) = P(**A**).

29. Assume **A**′∩**B**′ = ϕ. Show P(**A**) + P(**B**) = 1 + P(**A**∩**B**).

30. Give an example of a sample space, and events **A**, **B** where **A**′∩**B**′ = ϕ.